Matrices

A **matrix** is a rectangular array of numbers. For example, the following rectangular arrays of numbers are matrices:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 4 & 7 \\ 5 & 8 & 10 \end{bmatrix} \qquad C = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \\ 10 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 3 & 5 & 7 & 9 \end{bmatrix} \qquad E = \begin{bmatrix} 4 \\ 1 \\ 47653 \end{bmatrix}$$

Matrices vary in **size**. An $m \times n$ matrix has m rows and n columns. The matrices above have sizes

$$2 \times 2, \quad 2 \times 3, \quad 5 \times 1, \quad 1 \times 5, \quad 3 \times 1$$

respectively.

The numbers in the matrix are called the **entries** of the matrix. Because we may have the same number in more than one position, when we refer to an entry we refer to its position. The (i, j) entry is the entry in the ith row and jth column or the symbol A_{ij} denotes the entry in the i th row and j th column of the matrix A.

Example   Using the matrices shown above:

$$A_{12} = 2, \quad A_{21} = 3, \quad C_{31} = 6, \quad B_{23} = 10, \quad E_{31} = 47653.$$

Example   Using the matrices defined above find:

$$A_{22} =$$

$$B_{12} =$$

$$D_{13} =$$

$$E_{21} =$$

Matrices arise naturally in many areas of mathematics. We will use matrix notation to write systems of linear equations as a single matrix equation and we will also use matrix multiplication and inverse matrices to solve systems of equations when appropriate. The array format also allows us to list all possibilities compactly in situation where we have cross classification in probability theory. This will be especially useful in game theory.

Algebra of Matrices

Matrices have arithmetic properties, just like ordinary numbers. We can define an addition and a multiplication for matrices. In both of these binary operations there will be compatibility restrictions on the sizes of the matrices involved.

Adding Two matrices

Before we can add two matrices, they must have the same size. Any two matrices of the same size can be added. We add matrices by adding the corresponding entries. For example:

$$\begin{bmatrix} 2 & 1 & 0 \\ 4 & 0 & 1 \\ 1 & 2 & 3 \\ 0 & 3 & 10 \end{bmatrix} + \begin{bmatrix} 0 & 5 & 7 \\ 4 & 3 & 1 \\ 2 & 2 & 1 \\ 9 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 2+0 & 1+5 & 0+7 \\ 4+4 & 0+3 & 1+1 \\ 1+2 & 2+2 & 3+1 \\ 0+9 & 3+4 & 10+0 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 7 \\ 8 & 3 & 2 \\ 3 & 4 & 4 \\ 9 & 7 & 10 \end{bmatrix}$$

So, if we add two matrices, A and B, the (i,j) entry of $A+B$ is equal to $A_{ij} + B_{ij}$.

Example Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ $D = \begin{bmatrix} 1 & 0 \\ 5 & 9 \end{bmatrix}$

Then $A + D =$

Matrix Multiplication (A mild form)

We start by multiplying a row matrix by a column matrix. Here the number of entries in the row must equal the number of entries in the column. The general formula is given by

$$\begin{bmatrix} a_1 & a_2 & \cdots & a_{n-1} & a_n \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix} = a_1 \cdot b_1 + a_2 \cdot b_2 \cdots a_{n-1} \cdot b_{n-1} + a_n \cdot b_n$$

Example Calculate the following:

$$\begin{bmatrix} 1 & 2 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \\ 5 \\ 3 \end{bmatrix} = \qquad\qquad \begin{bmatrix} 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} =$$

$$\begin{bmatrix} 3 & 8 & 7 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 5 \end{bmatrix} = \qquad\qquad \begin{bmatrix} 1 & 8 & 4 & 5 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \\ 1 \\ 1 \end{bmatrix} =$$

Note that when you multiply a row by a column, you just get a number or a 1×1 matrix. Above we have (in order),

A 1×5 matrix multiplied by a 5×1 matrix gives a 1×1 matrix.
A 1×3 matrix multiplied by a 3×1 matrix gives a 1×1 matrix.
A 1×4 matrix multiplied by a 4×1 matrix gives a 1×1 matrix.
A 1×6 matrix multiplied by a 6×1 matrix gives a 1×1 matrix.

Compatibility   In order to multiply two matrices, we must have compatible sizes. Let A be an $m \times p$ matrix and let B be a $q \times n$ matrix. Then I can form the product AB only if $p = q$. If $p = q$, then AB will be an $m \times n$ matrix.

Example: let:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 4 & 7 \\ 5 & 8 & 10 \end{bmatrix} \qquad C = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \\ 10 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 3 & 5 & 7 & 9 \end{bmatrix} \qquad E = \begin{bmatrix} 4 \\ 1 \\ 47653 \end{bmatrix}$$

Which of the following matrix products can be formed and if it can be formed, what size is the matrix?

Product	AB	BC	AC	DC	AE	EA	EB	BE
Possible Y/N								
Size								

Rather than study general matrix multiplication, we will limit our study of matrix multiplication to that which will occur in game theory. Our goal is to be able to calculate products of the form:

$$AB \qquad BC \qquad \text{and} \qquad ABC,$$

where A is a $1 \times m$ row matrix, B is an $m \times n$ matrix and C is a column matrix of the form $n \times 1$. Because of associativity of matrix multiplication, the latter product ABC can be calculated in either of two ways, as $(AB)C$ or as $A(BC)$.

To multiply the $1 \times m$ row matrix A by the $m \times n$ matrix B, we multiply the row matrix A by the columns of B to get the entries of AB. Specifically, AB is a $1 \times n$ matrix (a row matrix) the $(1, j)$ entry of AB is the row matrix A multiplied by the j th column of B.

Example   Let $A = \begin{bmatrix} 1 & 2 \end{bmatrix}$ $\qquad B = \begin{bmatrix} 2 & 4 & 7 \\ 5 & 8 & 10 \end{bmatrix}$

Since A is a 1×2 matrix and B is a 2×3 matrix, AB will be a 1×3 matrix.

To calculate the $(1, 1)$ entry of AB, we multiply the row matrix A by column 1 of B

$$\begin{matrix} A & B & & AB \\ \begin{bmatrix} 1 & 2 \end{bmatrix} & \begin{bmatrix} 2 & 4 & 7 \\ 5 & 8 & 10 \end{bmatrix} & = & \begin{bmatrix} 1 \cdot 2 + 2 \cdot 5 & - & - \end{bmatrix} = \begin{bmatrix} 12 & - & - \end{bmatrix} \end{matrix}$$

To calculate the $(1, 2)$ entry of AB, we multiply the row matrix A by Column 2 of B.

$$\begin{matrix} A & B & & AB \\ \begin{bmatrix} 1 & 2 \end{bmatrix} & \begin{bmatrix} 2 & 4 & 7 \\ 5 & 8 & 10 \end{bmatrix} & = & \begin{bmatrix} 12 & 1 \cdot 4 + 2 \cdot 8 & - \end{bmatrix} = \begin{bmatrix} 12 & 20 & - \end{bmatrix} \end{matrix}$$

To calculate the $(1, 3)$ entry of AB, we multiply Row 1 of A by Column 3 of B.

$$\begin{matrix} A & B & & AB \\ \begin{bmatrix} 1 & 2 \end{bmatrix} & \begin{bmatrix} 2 & 4 & 7 \\ 5 & 8 & 10 \end{bmatrix} & = & \begin{bmatrix} 12 & 20 & 1 \cdot 7 + 2 \cdot 10 \end{bmatrix} = \begin{bmatrix} 12 & 20 & 27 \end{bmatrix} \end{matrix}$$

Example   Let

$$A = \begin{bmatrix} 1 & 2 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 1 \\ 0 & 2 \\ 4 & 1 \end{bmatrix}$$

Calculate AB

To multiply the $m \times n$ matrix B by the $n \times 1$ column matrix C, we multiply each row of B by the column matrix C to get the rows of BC. In particular, BC is a $m \times 1$ column matrix where the $(k, 1)$ entry of BC is the kth row of A multiplied by the column matrix B.

Example Let $B = \begin{bmatrix} 2 & 4 & 7 \\ 5 & 8 & 10 \end{bmatrix}$ and $C = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

To calculate the $(1, 1)$ entry of BC, we multiply row 1 of B by the column matrix C.

$$\begin{array}{cccc} B & C & & BC \\ \begin{bmatrix} 2 & 4 & 7 \\ 5 & 8 & 10 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} & = & \begin{bmatrix} 2 \cdot 1 + 4 \cdot 0 + 7 \cdot (-1) \\ - \end{bmatrix} = \begin{bmatrix} -5 \\ - \end{bmatrix} \end{array}$$

To calculate the $(2, 1)$ entry of BC, we multiply row 2 of B by the column matrix C.

$$\begin{array}{cccc} B & C & & BC \\ \begin{bmatrix} 2 & 4 & 7 \\ 5 & 8 & 10 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} & = & \begin{bmatrix} \overset{-5}{5 \cdot 1 + 8 \cdot 0 + 10 \cdot (-1)} \end{bmatrix} = \begin{bmatrix} -5 \\ -5 \end{bmatrix} \end{array}$$

Example Let $B = \begin{bmatrix} 3 & 1 \\ 0 & 2 \\ 4 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

Calculate BC.

<div align="center">

Converting a system of Linear Equations to a Matrix Equation

</div>

Given a system of linear equations in two unknowns

$$\begin{cases} -2x + 4y & = & 2 \\ -3x + 7y & = & 7 \end{cases}$$

we can write it in matrix form as a single equation $AX = B$, where

$$A = \begin{pmatrix} -2 & 4 \\ -3 & 7 \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \end{pmatrix}, \quad B = \begin{pmatrix} 2 \\ 7 \end{pmatrix}.$$

When we multiply we get

$$AX = \begin{pmatrix} -2x + 4y \\ -3x + 7y \end{pmatrix}$$

a 2×1 matrix. When we set this equal to the matrix $B = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$, we get our two equations back again by equating the elements of each matrix, thus getting our linear system back again:

$$\begin{cases} -2x + 4y &=& 2 \\ -3x + 7y &=& 7 \end{cases}$$

Example Convert the system of linear equations shown below to a matrix equation of the form $AX = B$.

$$\begin{cases} 2x_1 + 4x_2 &=& 2 \\ 3x_1 + 0x_2 &=& 2 \end{cases}$$

The same approach can be used for systems of equations with any number of variables. In this case, the system of n equations

$$\begin{cases} a_{1,1}x_1 + a_{1,2}x_2 + \cdots + a_{1,n}x_n &=& b_1 \\ a_{2,1}x_1 + a_{2,2}x_2 + \cdots + a_{2,n}x_n &=& b_2 \\ \vdots & & \vdots \\ a_{m,1}x_1 + a_{m,2}x_2 + \cdots + a_{m,n}x_n &=& b_m \end{cases}$$

can be written as a matrix equation $AX = B$ where

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n} \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, \quad B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

Example Convert the system of linear equations shown below to a matrix equation of the form $AX = B$.

$$\begin{cases} 2x_1 + 4x_2 + 3x_3 + x_4 &=& 2 \\ x_1 + x_2 + 0x_3 + 2x_4 &=& 4 \\ 0x_1 + x_2 + x_3 + 0x_4 &=& 5 \\ 3x_1 + 0x_2 + x_3 + 2x_4 &=& 2 \end{cases}$$

Multiplication and multiplicative inverses for square matrices

If a matrix has the same number of rows as it does columns, it is called a square matrix. We will see below that the set/system of all $n \times n$ matrices has all of the characteristics of the real number system that are necessary to perform basic algebra to solve for unknown variables. We will use this algebra to solve matrix equations resulting from a system of equations.

Example The 2×2, 3×3 and 4×4 matrices shown below are square matrices.

$$\begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix} \qquad \begin{bmatrix} 2 & 1 & 0 \\ 4 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix} \qquad \begin{bmatrix} 2 & 1 & 0 & 5 \\ 4 & 0 & 1 & 2 \\ 1 & 2 & 3 & 9 \\ 0 & 3 & 10 & 0 \end{bmatrix}$$

Note that for any given n, we can add, subtract and multiply two $n \times n$ matrices to get another $n \times n$ matrix. The zero $n \times n$ matrix (an $n \times n$ matrix where all entries are 0) which we denote by $0_{n \times n}$ has the properties that

$$A + 0_{n \times n} = A - 0_{n \times n} = A$$

and

$$A - A = 0_{n \times n}$$

for any $n \times n$ matrix A. Thus $0_{n \times n}$ plays the same role in the system of $n \times n$ matrices as that played by the number 0 in the real number system.

Algebra with matrices   This allows us for example to solve for X (a 2×2) matrix in the following equation as we would in regular algebra:

$$\begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix} + X = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$$

We subtract the matrix $\begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix}$ from both sides to get

$$\begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix} + X - \begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix}$$

this gives

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + X = \begin{bmatrix} -2 & 0 \\ -1 & 2 \end{bmatrix} \quad \text{and thus} \quad X = \begin{bmatrix} -2 & 0 \\ -1 & 2 \end{bmatrix}$$

Indentity Matrix   In the real number system, the number 1 is to multiplication as the number 0 is to addition. We also have an analogue of the number 1 in an $n \times n$ matrix system which we denote by $I_{n \times n}$. It is the matrix with 1's on the main diagonal and 0's elsewhere. We show $I_{2 \times 2}$, $I_{3 \times 3}$ and $I_{4 \times 4}$ below.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For an $n \times n$ matrix A, we have

$$A \cdot I_{n \times n} = A = I_{n \times n} \cdot A.$$

Inverse Matrices and Determinants   In the real number system, division is to multiplication what subtraction is to addition. However, we have some restrictions on division in that we can never divide

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by the number 0. We also have division in an $n \times n$ matrix system with some restrictions. For each $n \times n$ matrix, there is a number associated to the matrix called the determinant. So for an $n \times n$ matrix A, we denote the **determinant by det$A$.** We can only divide by matrices for which det$A \neq 0$. We will show how to calculate this determinant for a 2×2 matrix below. The formula is more complicated for $n \times n$ matrices with $n \geq 3$ and we will use Mathematica to calculate determinants when necessary.

Determinant and Inverse of a 2×2 matrix For a two by two matrix, $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, we have det$A = ad - bc$. If this number is non-zero, we can create a matrix which "cancels" the matrix A when multiplied by A, namely the matrix

$$\left(\frac{I}{A} = \right) \quad A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad \text{ - } bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

This inverse matrix has the following properties:

$$A \cdot A^{-1} = I_{2 \times 2} = A^{-1} \cdot A.$$

Example Let $A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$. Find det A, determine if A has an inverse and if so find it.

Using matrix inverses and Mathematica to solve systems of equations
(Using 2.4, Goldstein, Schneider and Siegel and Mathematica(available on the OIT website))

Similarly, if A is an $n \times n$ matrix with det$A \neq 0$, we can create an inverse matrix A^{-1} with the properties

$$A \cdot A^{-1} = I_{n \times n} = A^{-1} \cdot A.$$

Canceling matrices in Matrix Equations When solving for the unknown x in the equation $2x = 4$ where x is a real number, we divide across by 2 (multiply by $\frac{1}{2}$) to cancel the 2; we get

$$\frac{1}{2} \cdot 2x = \frac{1}{2} \cdot 4 \quad \text{which gives} \quad x = 2.$$

Similarly, if we have a matrix equation, $AX = B$, where A is an $n \times n$ matrix, X and B are $n \times m$ matrices and set $A \neq 0$, we can solve for the unknown matrix X in the matrix equation

$$AX = B$$

by multiplyingg both sides by A^{-1} to get

$$A^{-1}AX = A^{-1}B \quad \text{which gives} \quad I_{n \times n}X = A^{-1}B \quad \text{which in turn gives} \quad X = A^{-1}B.$$

Example Solve the system of equations shown below by solving the corresponding matrix equation.

$$\begin{array}{rcrcl} 2x_1 & + & x_2 & = & 3 \\ x_1 & + & x_2 & = & 5 \end{array}$$

If a system of linear equations has matrix form $AX = B$, where A is an invertible $n \times n$ matrix, then there is a unique solution to the system of equations. That solution is given by $X = A^{-1}B$. To solve a matrix equation of the form $AX = B$ where A is an invertible $n \times n$ matrix, we will use Mathematica.

Mathematica

You may and should download your own free copy of *Mathematica* from the OIT website Go to the OIT homepage

http://oit.nd.edu/

click on `Software Downloads` in the menu on the left.

Enter your Notre Dame netID and Password in the fields provided

Look for `Mathematica` on the list of available software

Click on `More Details` and follow the directions to download the Student Version.

When you open *Mathematica*, you should open a new Notebook by clicking on `File` in the menu bar, then follow the path `File/New/Notebook`. You can also create a new Notebook, by pressing **Command-n** on your keyboard.

To enter a command, click on the plus mark in the upper left of your document and choose `Mathematica input` from the drop down menu. Note that this is also the default mode, so you can just start typing your command without this step, if you click below the plus mark.

type `2 + 3`

and press `Shift-Return` to get your answer 5 on a new line.

Another way to Evaluate a command in an input cell is to click anywhere on the code in the cell, go to `Evaluation` in the menu at the top of your screen and choose `Evaluate Cells` from the drop down menu.

To solve a system of linear equations in `Mathematica`, we will use their matrix form.

We first learn how to **enter a matrix** and give it a name. We represent the matrix as a list of rows separated by commas between curly brackets. Each row is enclosed by curly brackets with commas separating the entries.

Example The matrix

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 1 & 5 & 10 \end{bmatrix}$$

should be entered as
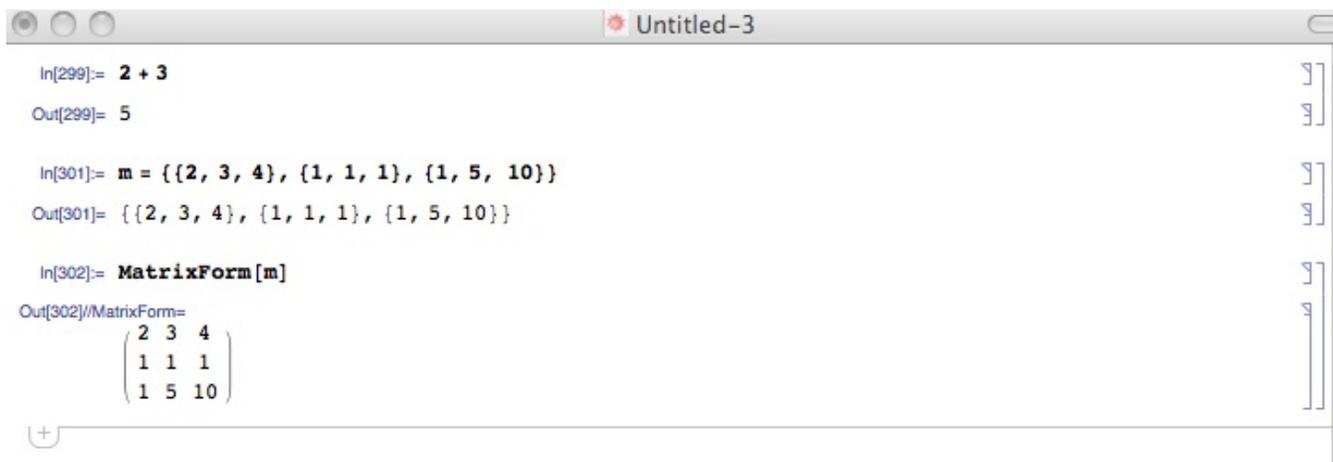
$$\{\{2, 3, 4\}, \{1, 1, 1\}, \{1, 5, 10\}\}$$

When you enter this matrix as input, press `Shift-return` to get

The output, looks exactly like the input. if you prefer the usual matrix format, we will discuss how to view that below. If we want to use this matrix in calculations, we must be able to refer to it, so we must give it a name. To call the matrix m, we can go back to the input cell and simply insert $m =$ in front of our matrix. Then we press `Shift-Return` to get the same output, except now Mathematica knows that this matrix is called m. If you wish to view your matrix in the usual format, you can type the command `MatrixForm[m]` and Mathematica will present the matrix in its usual form.

Now suppose we wanted to solve for X in the matrix equation

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 1 & 5 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \\ 16 \end{bmatrix},$$

then we need to solve for the matrix X in the equation

$$mX = \begin{bmatrix} 9 \\ 3 \\ 16 \end{bmatrix}.$$

Lets call the matrix on the right b and enter it into Mathematica. Note there is just one entry in each row.

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```
In[301]:= m = {{2, 3, 4}, {1, 1, 1}, {1, 5, 10}}

Out[301]= {{2, 3, 4}, {1, 1, 1}, {1, 5, 10}}

In[302]:= MatrixForm[m]

Out[302]//MatrixForm=
        ⎛ 2  3  4  ⎞
        ⎜ 1  1  1  ⎟
        ⎝ 1  5  10 ⎠

In[303]:= b = {{9}, {3}, {16}}

Out[303]= {{9}, {3}, {16}}

In[304]:= MatrixForm[b]

Out[304]//MatrixForm=
        ⎛ 9  ⎞
        ⎜ 3  ⎟
        ⎝ 16 ⎠
```

Now to solve the system, we must solve for the matrix X the matrix equation $mX = b$. To do this in Mathematica, we use the command `LinearSolve[m,b]`.

```
In[301]:= m = {{2, 3, 4}, {1, 1, 1}, {1, 5, 10}}

Out[301]= {{2, 3, 4}, {1, 1, 1}, {1, 5, 10}}

In[302]:= MatrixForm[m]

Out[302]//MatrixForm=
        ⎛ 2  3  4  ⎞
        ⎜ 1  1  1  ⎟
        ⎝ 1  5  10 ⎠

In[303]:= b = {{9}, {3}, {16}}

Out[303]= {{9}, {3}, {16}}

In[304]:= MatrixForm[b]

Out[304]//MatrixForm=
        ⎛ 9  ⎞
        ⎜ 3  ⎟
        ⎝ 16 ⎠

In[305]:= LinearSolve[m, b]

Out[305]= {{1}, {1}, {1}}
```

The output is the matrix X, we can view this matrix in matrix format if we change our command to `MatrixForm[LinearSolve[m,b]]`. We se that

$$X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

To save your file, use `command-S`. The first time you save the file, you will need to supply a name and choose where to save it.

Example   Convert the system of linear equations shown below to a matrix equation of the form $AX = B$ and solve the system using Mathematica.

$$\begin{cases} x + 2y + z & = & 2 \\ x - y + 2z & = & 4 \\ x - y - z & = & 5 \end{cases}$$

Example Convert the system of linear equations shown below to a matrix equation of the form $AX = B$ and solve the system using Mathematica.

$$\begin{cases} 2x_1 + 4x_2 + 3x_3 + x_4 & = & 2 \\ x_1 + x_2 + 0x_3 + 2x_4 & = & 4 \\ 0x_1 + x_2 + x_3 + 0x_4 & = & 5 \\ 3x_1 + 0x_2 + x_3 + 2x_4 & = & 2 \end{cases}$$